BESSEL CALCULATIONS for FM....
or How to REALLY Impress Your Friends!

by James K. Thorusen KCBS-TV Transmitter

This section deals with the calculation of suitable tone frequencies for the purpose of producing a carrier null in an FM transmitter at a particular value of deviation.

First, some rules:

Rule 1: Carrier vanishes when $\Delta\,\theta$ (radians) = 2.40 and 5.52 and 8.65 + n π.

Rule 2: $\Delta\,\theta$ (radians) = Modulation Index.

Rule 3: Modulation Index (MI) = $D \div f_a$ where:

D = Deviation

f_a = Frequency of applied Audio (tone).

Therefore, the carrier vanishes when $D \div fa = 2.40$ and 5.52 and 8.65 + n π

So, to calculate a Bessel null tone for any given application, we need only know the above plus the desired deviation. We then plug the numbers into the formula for Modulation Index, do a bit of algebra, and we are there.

$D \div f_a = 2.40$ (first null).... so...... $D = (2.40) \bullet f_a$ and.... $D \div 2.40 = f_a$

Example 1: Find the first Bessel null tone for setting TV aural deviation to the mandated 25 KHz.

Solution:

$25\text{ KHz} \div f_a = 2.40$

$25\text{ KHz} = (2.40) \bullet f_a$

$25\text{ KHz} \div 2.40 = f_a =$ **10.416 KHz**.

Example 2: Find a **SUITABLE** Bessel null tone for setting FM broadcast deviation to the required 75 KHz.

Solution: 75 KHz ÷ fa = 2.40 (first Bessel null)

fa = 31.25 KHz.

Although 31.25 KHz. would produce a null, it is not a suitable tone for this application. The reason for this is that 31.25 KHz. is well outside the normal audio passband of FM broadcast equipment, which normally cuts off at 15 KHz. Therefore, a lower frequency tone is necessary. Lower frequency tones can be selected to produce nulls of the carrier at the required deviation value, but they are second or third nulls (MI of 5.52 or 8.65 + n π), rather than the first null (MI of 2.40) of example 1.

Therefore: 2nd null (from Rule 1) = MI of 5.52

so... 75 KHz. ÷ f_a = 5.52

f_a = 13.587 KHz.

Note that it is necessary to go THROUGH the first null to reach the correct deviation value which is represented by the SECOND null. In other words, when using a larger Modulation Index value, the first Bessel null will be reached at a lower deviation than the desired value of 75 KHz. In our example above, the first null would be reached at a deviation of 32.6 KHz.

D ÷ 13.587 KHz. = 2.40

Where:
13.587 KHz. = Bessel tone for MI of 5.52 calculated above
and... 2.40 = MI for first null

so... 2.40 • (13.587 KHz.) =D

Therefore.... D = 32.6 KHz.

Example 2 continued:

Remember, when using higher order nulls, it is necessary to increase the deviation adjustment from zero past the first carrier null and continue increasing deviation until the carrier nulls again for the nth time, where n is the number of the null chosen from the Bessel values in Rule 1, i.e. for MI of 2.40, n = 1; for MI of 5.52, n =2; for MI of 8.65, n = 3; for MI of 8.65 + π, n = 4, for 8.65 + 2 π, n = 5, etc.

As a general rule, it is advisable to use the lowest value of n that will allow the measurement to be made. Higher values of n than 2 or 3 may produce very poor nulls, or no visible nulls at all, if there is any distortion or noise present in the system.

BESSEL CALCULATIONS for FM....
or How to REALLY Impress Your Friends!

by James K. Thorusen KCBS-TV Transmitter

This section deals with how to calculate the sideband levels that one would observe on a spectrum analyzer for a given modulation index.... that is, for given values of deviation and modulating frequency. This is useful for determining the injection level of subcarriers, or the deviation of a single tone modulating an FM transmitter if the frequency of the tone and the sideband power are known.

The first example will be the determination of the correct injection level of the stereo pilot in the BTSC stereo for analog TV modulation scheme.

First, some rules:

Rule 1. The entries in the tables of Bessel functions correspond to VOLTAGE ratios of the sidebands or carrier WITH MODULATION to the UNMODULATED carrier.

Rule 2. The **"Z"** entries in the tables correspond to the modulation index.

Rule 3. The Modulation Index is defined as: Deviation ÷ Frequency of Modulation.

Rule 4. The Modulation Index is interpolated in the Bessel Function table as follows:

- Rule 4A. The whole number (integer) part of the Modulation Index number is found on the left-most vertical COLUMN of the table.
- Rule 4B. The (decimal) fractional part of the Modulation Index number is found on the top ROW of the table.
- Rule 4C. Go to the row corresponding to the whole number part of the Modulation Index. Scroll across that row to the column under the fractional part of the Modulation Index number. The number found at this intersection of row and column is the voltage ratio for this modulation index.

Rule 5. The $J_0(z)$ Bessel function table describes the CARRIER voltage.

Rule 6. The $J_1(z)$ Bessel function table describes the **FIRST** SIDEBAND voltage.

Rule 7. It may be necessary to further interpolate the Bessel function table in order to achieve the necessary accuracy. Examine the table entries near the nearest tenth value that you intend to use. If the values change less than 10 % from the previous tenth to the nearest tenth, then .1 accuracy is probably sufficient. However, if the change is greater than 10%, then further interpolation will probably be necessary. (See the examples below.)

Rule 8. To calculate the amount by which the first sideband power should be below the carrier power for a given Modulation Index, the voltage ratios from the two Bessel Function tables [$J_0(z)$ and $J_1(z)$] must be inserted into the dB formula for VOLTAGE, thus: $dB = 20 \bullet \log_{10} (V_1 \div V_2)$ where V1 and V2 are the two voltage values obtained from the Bessel Function Tables.

Note: All of the following examples (except Example 4) are set up assuming a single measurement of the MODULATED carrier and the level of the sidebands with respect to it. If it is possible to remove all modulation from the carrier and note the carrier level in the total absence of all modulation, then a value of 1 may be substituted in all of the following equations instead of the table value for $J_0(z)$ which will simplify the calculations and the table usage somewhat. If this technique is used, then the dB value that is obtained will be the number of dB below the UNMODULATED carrier level, and not the carrier as observed on the spectrum analyzer with modulation present.

BESSEL CALCULATIONS for FM....
or How to REALLY Impress Your Friends!
by James K. Thorusen KCBS-TV Transmitter

Example 1: Suppose we wish to verify the magnitude of the subcarrier injection of the BTSC Stereo Pilot in a TV aural transmitter.

First, Givens: Stereo Pilot Frequency = 15.734 KHz. (From BTSC standards documents).
Stereo Pilot Injection Level = 5 KHz. (Ibid).

Second, Calculate Modulation Index:

MI = 5 ÷ 15.734 = .318

Third, Look up Δ CARRIER voltage in table $J_0(z)$: For Modulation Index of 0.3, the voltage ratio of the modulated carrier to the unmodulated carrier is .9776. The changes from modulation indices of .2 to .3 to .4 are less than 2%, so .318 is close enough to .3; no further interpolation is necessary.

Forth, Look up Δ **FIRST** SIDEBAND voltage in table $J_1(z)$: For Modulation Index of 0.3, the voltage ratio of the **FIRST** SIDEBAND to the unmodulated carrier is .1483. However, the changes from modulation indices of .2 to .3 to .4 in this table are significant.... on the order of 30% . Therefore, further interpolation will be necessary. The changes from .2 to .3 to .4 are roughly the same; the Bessel function curve is fairly linear at this point, so we can interpolate between the table entries for .3 and .4, which is .1960. The change between modulation indices of .3 and .4 is .0477, which we can round to .05. The actual modulation index in our problem is .318, which is about .2 of the way between the table values of .3 and .4. So, we can add .2 of .05, or .01 to our table value, which gives us a value of .1583 for our modulation index of .318.

Example 1 continued.

Fifth, plug these numbers into the dB formula:

$dB = 20 \bullet \log_{10} (.1583 \div .9776) = -15.8$ dB.

In other words, for a pilot injection of 5 KHz., the first sidebands on a spectrum analyzer display (assuming no other modulation) should be 15.8 dB below the modulated carrier. Note that the number in the $J_1(z)$ (First sideband) table represents a value for the voltage ratio of the sideband to the UNMODULATED carrier. Since the carrier power decreases slightly with modulation (the energy goes into the sidebands) it is necessary to look up the change in carrier voltage also and compare that to the first sideband voltage, rather than a value of 1 for unmodulated carrier.

Example 2: Calculate the sideband power levels for the 5 KHz. pilot in example 1 for the maximum and minimum permitted levels of pilot injection (deviation): 4.5 KHz. and 5.5 KHz.

First, Givens: Stereo Pilot Frequency = 15.734 KHz. (From BTSC standards documents; same as in example 1 above.).

Stereo Pilot Injection Level = 4.5 KHz. (Minimum permitted value).

Second, Calculate Modulation Index:

$MI = 4.5 \div 15.734 = .286$

Third, Look up Δ CARRIER voltage in table $J_0(z)$: For a Modulation Index of 0.3, the voltage ratio of the modulated carrier to the unmodulated carrier is .9776. The changes from modulation indices of .2 to .3 to .4 are less than 2%, so .286 is close enough to .3; no further interpolation is necessary.

Forth, Look up Δ **FIRST** SIDEBAND voltage in table $J_1(z)$: For a Modulation Index of 0.3, the voltage ratio of the **FIRST** SIDEBAND to the unmodulated carrier is .1483. However, the changes from modulation indices of .2 to .3 to .4 in this table are significant.... on the order of 30%. Therefore, further interpolation will be necessary. The changes from .2 to .3 to .4 are roughly the same; the Bessel function curve is fairly linear at this point, so we can interpolate between the table entries for .3 and .2, which is .0995. The change between modulation indices of .3 and .2 is .0488, which we can round to .05. The actual modulation index in our problem is .286, which is about .9 of the way between the table values of .2 and .3. So, we can add .9 of .05, or .045 to our table value of .0995, which gives us a value of .1445 for our modulation index of .286.

Example 2 continued.

Fifth, plug these numbers into the dB formula:

dB = 20 • $\log_{10}$ (.1445 ÷ .9776) = -16.6 dB.

Therefore, the lower limit of pilot injection is represented by first sidebands 16.6 dB below the carrier.

Next, we calculate the upper limit, as follows:

First, Givens: Stereo Pilot Frequency = 15.734 KHz. (From BTSC standards documents; same as in example 1 above.).

Stereo Pilot Injection Level = 5.5 KHz. (Maximum permitted value).

Second, Calculate Modulation Index:

MI = 5.5 ÷ 15.734 = .35

Third, Look up Δ CARRIER voltage in table $J_0(z)$: For a Modulation Index of 0.4, the voltage ratio of the modulated carrier to the unmodulated carrier is .9604. The changes from modulation indices of .2 to .3 to .4 are less than 2%, so .35 is close enough to .4; no further interpolation is necessary.

Example 2 continued.

Forth, Look up Δ **FIRST** SIDEBAND voltage in table $\boldsymbol{J_1(z)}$: For a Modulation Index of 0.3, the voltage ratio of the **FIRST** SIDEBAND to the unmodulated carrier is .1483. However, the changes from modulation indices of .2 to .3 to .4 in this table are significant.... on the order of 30%. Therefore, further interpolation will be necessary. The changes from .2 to .3 to .4 are roughly the same; the Bessel function curve is fairly linear at this point, so we can interpolate between the table entries for .3 and .4, which is .1960. The change between modulation indices of .3 and .4 is .0477, which we can round to .05. The actual modulation index in our problem is .35, which is about .5 of the way between the table values of .3 and .4. So, we can add .5 of .05, or .025 to our table value of .1483, which gives us a value of .1733 for our modulation index of .35.

Fifth, plug these numbers into the dB formula:

$dB = 20 \bullet \log_{10} (.1733 \div .9776) = -15$ dB.

Therefore, the upper limit of pilot injection is represented by first sidebands 15 dB below the carrier.

Example 3.

Calculate the first sideband power levels for the 19 KHz. pilot for FM stereo broadcast.

First, Givens: Stereo Pilot Frequency = 19 KHz. (From FCC standards).
Stereo Pilot Injection Level = 7.5 KHz. (Maximum permitted value).

Second, Calculate Modulation Index:

MI = 7.5 ÷ 19 = .394

Third, Look up Δ CARRIER voltage in table $\boldsymbol{J_0(z)}$: For a Modulation Index of 0.4, the voltage ratio of the modulated carrier to the unmodulated carrier is .9604. The changes from modulation indices of .2 to .3 to .4 are less than 2%, so .394 is close enough to .4; no further interpolation is necessary.

Forth, Look up Δ **FIRST** SIDEBAND voltage in table $\boldsymbol{J_1(z)}$: For a Modulation Index of 0.4, the voltage ratio of the **FIRST** SIDEBAND to the unmodulated carrier is .1960. The changes from modulation indices of .2 to .3 to .4 in this table are significant.... on the order of 30%. Normally, interpolation would be necessary. However, the actual modulation index in our problem is .394, which is very close to .4, so we can use the .4 value, or .1960.

Fifth, plug these numbers into the dB formula:

$dB = 20 \bullet \log_{10} (.1960 \div .9604) = -13.8$ dB.

Therefore, the upper limit of pilot injection is represented by first sidebands 13.8 dB below the carrier.

Example 4. Given the carrier level WITHOUT modulation, the modulating frequency, and the number of dB below the UNMODULATED carrier that is represented by the first sidebands, calculate the deviation (injection) of the BTSC pilot from examples 1 and 2.

Givens: Carrier level with NO modulation applied = 1. (Always normalize to 1 for purposes of this computation, since the Bessel tables are all ratios with 1 in the denominator.)

Note: It is necessary to measure the power of the unmodulated carrier and normalize it to a value of 1 to perform this calculation. This is because of the fact that when working the calculation backwards, it is necessary to have a known value for V_1. Otherwise, the problem becomes one of 1 equation in 2 unknowns, which can have a large number of possible solutions.

dB below UNMODULATED carrier of first sidebands = 15.8 dB.
Frequency of modulation = 15.734 KHz.

First, we need to do some algebra. Since dB = $20 \bullet \log_{10} (V_1 \div V_2)$ we can work the formula backwards and solve for V_2, thus: First, we rewrite the formula so that we have the given sideband value (we will assign S to this variable) and 1 (the normalized carrier voltage) in our formula: $S = 20 \bullet \log_{10} (1 \div V_2)$. Then we get rid of the 20: $S \div 20 = \log_{10} (1 \div V_2)$. Then we take the antilog, which gets rid of the $\log_{10}$ on the right side, and simply means raising 10 to the $S \div 20$ power on the left side, thus: $10^{(S \div 20)} = 1 \div V_2$. Next, we get rid of V_2 in the denominator on the right side by multiplying both sides by V_2 thus: $V_2 \bullet (10^{(S \div 20)}) = 1$. We need V_2 by itself, so we divide both sides by $(10^{(S \div 20)})$, giving: $V_2 = 1 \div (10^{(S \div 20)})$.

Example 4 continued:

Plugging in our givens produces the following formula: $V2 = 1 \div 10^{(15.8 \div 20)}$,

or $V_2 = 1 \div 10^{(.79)}$ or $V_2 = 1 \div 6.17$ or $V_2 = .162$.

Since this is our voltage ratio with respect to the unmodulated carrier (which we have normalized to 1), we can now look this number up directly on the Bessel $\boldsymbol{J_1(z)}$ table.

(Note: because the Bessel function is a repeating function, you must have some idea of the deviation to begin with, so that you will be on the correct row in the table. You can calculate a rough maximum and minimum MI based on worst case "guestimates" of the deviation in question which should place you on the correct row of the table. If there are no numbers near the one you have calculated in the row you are using in the Bessel table, then either you have chosen the wrong row, your "guestimates" were too far off, or you have made an error in calculating the voltage ratio from the sideband level data.)

Unfortunately, the number we have calculated (.162) lies between .1483 (MI = .3) and .1960 (MI = .4) on the table, so we will have to interpolate. The difference between .1483 and .1960 is .0477, which we can round up to .05. The difference between .1483 and .162 (our calculated Bessel value) is .0137, which we can round to .014. The value .014 is .28 of .05, or .28 of the way between .1483 (MI = .3) and .1960 (MI = .4). Therefore, our Modulation Index must also be .28 of the distance between these values, (ie. .3 and .4). Since the distance between .3 and .4 is .1, this works out to be $.1 \bullet .28$ or .028. It is of course closer to an MI of .3 than an MI of .4 so we calculate an MI for our data by simply adding our .028 "distance between" value to the lower value of .3 (the MI for .1483), giving an MI of .328.

Example 4 continued:

Now that we have a Modulation Index, it becomes a simple matter to plug it into the following formula, together with the frequency of the modulation, and obtain the Deviation, or the injection of the modulation in KHz.

$MI = D \div F_a$ where: MI is the modulation index

D is the deviation

F_a is the modulating frequency.

So: $.328 = D \div 15.734$.

Multiplying both sides by 15.734, we get:

$.328 \bullet (15.734) = D$

or   D = 5.16 KHz. of Deviation or Injection, whichever you choose to call it.